An Integer Programming Model to Optimize Resource Allocation for Wildfire Containment

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ABSTRACT. Determining the specific mix of fire-fighting resources for a given fire is a necessary condition for identifying the minimum of the Cost Plus Net Value Change (*C+NVC*) function. Current wildland fire management models may not reliably do so. The problem of identifying the most efficient wildland fire organization is characterized mathematically using integer-programming techniques. This mathematical exposition is then solved using the LINGO optimization language. Sensitivity analysis is conducted on model inputs to demonstrate the flexibility of the model architecture. Further, the model is used to model budget constraints commonly faced by fire managers. FOR. SCI. 49(2):331–335.

Key Words: Linear programming, integer programming, wildfire management, optimization.

ECONOMIC THEORY has played a central role in wildland fire management since Headley (1916) and Sparhawk (1925) described trade-offs involved in establishing an optimal wildfire management program level (Pyne et al. 1996, p. 428). The theoretical framework used to identify the most economically efficient level of fire management expenditure has been the Cost plus Net Value Change model (*C+NVC*) (Gorte and Gorte 1979). This model minimizes the cost of wildfire by minimizing the sum of presuppression (expenditures on wildfire management prior to a fire season), suppression (direct wildfire suppression expenditures during a fire season), and *NVC* (net wildfire damages). While the *C+NVC* model, as typically illustrated (Pyne et al. 1996, p. 428), provides a theoretical framework for wildfire management, it does not specify which fire-fighting resources should be used to achieve the minimum value of *C+NVC*. A solution that is to have operational value must also indicate the specific mix (i.e., which fire-fighting resources are employed, not just total budgets) of fire-fighting resources to be employed for a given wildfire (Gonzalez-Caban 1986).

In response to a 1978 congressional mandate (Gorte and Gorte 1979) requiring cost-benefit analysis of future budget requests, the USDA Forest Service developed the National Fire Management Analysis System (NFMAS). NFMAS was the first operational model based on the *C+NVC* theoretical framework designed to solve for the most efficient mix of fire-fighting resources. The user selects specific fire-fighting resources, presuppression budgets, and dispatch rules[1], and tracks the resulting costs and damages for a given geographical area and set of fire behavior conditions. The user systematically changes these inputs to identify the mix of fire-fighting resources, and dispatch rules, which are intended to identify the minimum value of *C+NVC*. NFMAS is a simulation model that requires the user to make multiple runs to identify the optimal solution. Such an iterative approach has certain drawbacks (Donovan et al. 1999), principally that the optimal mix of fire-fighting resources may not be reliably identified.

While NFMAS is the most widely used economic fire management model for public lands in the United States, others not so closely tied to economic theory have been developed. For example, the CFES-IAM model (Fried and Gilless 1988) was developed for the California Division of Forestry. It does not directly consider the economic costs of

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wildfire damages, but rather implements a California legislative mandate to provide equal protection for lands of equal value. The National Park Service (NPS) uses a fire management model called FIREPRO (NPS 1997), which does directly consider resource values, nor was it designed to solve for the optimal mix of fire-fighting resources.

The fire management models described above are used for strategic fire management and determining fire organizations for an entire season with multiple fire events. This article develops a model to determine the optimal fire organization for a single historic fire. The purpose of the model is to demonstrate the feasibility of using optimization techniques for wildfire planning and budgeting; to suggest ways that this approach may be extended to more complex fire events.

The optimal fire organization problem is first characterized mathematically, using integer-programming techniques. LINGO[2], a linear and integer programming software package, is then used to solve the mathematical exposition. The integer program selects those fire-fighting resources that minimize the sum of all fire-related costs and damages. By identifying the optimal mix of fire-fighting resources, the model applies the theory of *C+NVC* to a meaningful fire management scenario. Sensitivity analysis is conducted by altering fire behavior and fire-fighting resource parameters, to demonstrate the ability of the integer-programming model to respond to differing model parameters. Further, the integer-programming model is used to solve for the optimal mix of fire-fighting resources while facing different types of budget constraints. This type of constrained optimization illustrates the model's capacity to accommodate realistic fire management constraints.

Methods

Problem Characterization

Defining the most economically efficient fire organization for a particular fire requires determining which resources should be dispatched in which time periods to contain (construct a line around) the fire at minimum cost (*C+NVC*). This constitutes an optimization problem that lends itself to Integer Programming (IP) because fire-fighting resources are indivisible units and are dispatched accordingly.

Determining the optimal fire organization has features in common with the well-characterized knapsack problem (Winston 1994, p. 468). The knapsack problem involves maximizing the benefit from the contents of a knapsack, given a range of possible items that can be selected. Each item has a defined benefit and weight, while the knapsack itself has a total weight limit. To aid in solving the problem, a binary decision variable is defined, which takes on a value of 1 if the item is selected, 0 otherwise: the variable $x_i = 1$ if the *i*th item is selected, and 0 otherwise.

As with the knapsack problem, the optimal fire organization problem involves optimizing an objective function by selecting from a menu of possible items, while subject to a constraint. The objective is to minimize the sum of fire-related costs and damages, and the selection is made from a menu of discrete fire-fighting resources, while the constraint is fire containment. The fire organization problem has an additional temporal dimension not present in the knapsack problem. In the knapsack problem, a resource is selected and used during only one time period. However, in the fire organization problem, once a resource has been selected it may be used during more than one time period.

The temporal dimension of the fire problem complicates the containment constraint. If the fire is contained in a given time period, then its final perimeter will be smaller than if the fire is contained in a later time period. Further, the containment constraint need only be fulfilled in one time period, as fire containment is assumed to result in fire control.

Each fire-fighting resource is defined as having four parameters: fixed rental cost, variable cost, arrival time, and rate of line production. The fixed rental charge represents the cost of renting a resource and is paid once if a resource is used in one or more time periods. In addition to a fixed rental charge, a variable cost must be paid for every time period in which a resource is used. This variable cost represents the hourly cost of operating a given resource. Arrival time is the period of time it takes a fire-fighting resource to travel to the fire; during this period it cannot produce line.

The following formulation mathematically characterizes the objective function and constraints that are required to identify the fire organization that minimizes the sum of all fire-related damages and costs. The model has *m* time periods and *n* fire-fighting resources.

$$MIN = \sum_{j=1}^{m}\sum_{i=1}^{n} C_i H_j D_{i,j} + \sum_{i=1}^{n} P_i Z_i + \sum_{j=1}^{m} NVC_j N_j \quad (1)$$

subject to

$$\sum_{i=1}^{n}\sum_{j=1}^{m} (H_j - A_i) PR_i D_{i,j} \geq \sum_{j=1}^{m} PER_j N_j \quad (2)$$

$$\forall_i \sum_{j=1}^{m} D_{i,j} \leq (Z_i) \quad (3)$$

$$\forall_j (SP_j N_j - L_j) \leq mn(Y_j) \quad (4)$$

$$\forall_j \sum_{i=1}^{n} (H_j - A_i) PR_i D_{i,j} = L_j \quad (5)$$

$$\forall_j N_{j+1} = Y_j \quad (6)$$

Decision Variables

$D_{i,j}$ Binary variable that takes on a value of 1 for the time period during which containment is achieved for resources employed.

L_j Total line construction up to and including a given time period in kilometers.

Y_j Binary variable defining whether the fire is uncontained in the jth time period. Uncontained = 1, contained = 0

N_j A one time period lagged variable on Y_j.

Z_j Binary variable defining whether the ith resource has been dispatched. Dispatched = 1, not dispatched = 0

Fire Behavior Parameters

PER_j Increment in fire perimeter growth for the jth time period in kilometers.

NVC_j Increment in net value change for the jth time period.

SP_j Total fire perimeter up to and including the jth time period in kilometers.

H_j Time period counter.

Fire-fighting Resource Parameters

C_i Hourly cost of operating the ith resource.

P_i Rental cost of the ith resource.

PR_i Line production rate of the ith resource in kilometers.

A_i Arrival time to the fire of the ith resource.

The objective function specifies that resources be dispatched to ensure that the sum of all costs and damages is minimized. The variable N_j ensures that increments of *NVC* are only included for time periods during which the fire is not contained, or is in the process of being contained (N_j is constrained to be zero for time periods during which the fire is fully contained). On containment, the fire is assumed to cause no further damage.

Inequality (2) requires that during one of the *m* time periods, total line construction must exceed total fire perimeter [NB, since the LHS of inequality (2) is constrained to be greater than or equal to the RHS, the LHS is constrained to be nonnegative]. If quickly dispatched resources are used, then the fire can be contained when its perimeter is smaller. If resources with longer arrival times are used, then the fire perimeter will be larger at containment. The variable N_j is used to model the changing fire perimeter over time. It is possible that for a given set of fire behavior characteristics, the fire-fighting resources employed would not be able to contain the fire in the time period allowed. In this case there would be no feasible solution. NFMAS deals with this problem by employing an escaped fire table that assigns damage to fires that are not contained during initial attack. We have decided not to account for escaped fires. Escaped fires are an important ancillary problem that is beyond the scope of our IP formulation.

Table 1. Fire growth characteristics.

Hours	Perimeter (km)	Area (ha)
1	0.3	0.7
2	1.0	5.6
3	1.3	9.6
4	1.8	15.9
5	2.0	20.3
6	2.2	24.3

Inequality (3) constitutes a conditional if/then constraint for Z_i. If a resource i is used during any time period, then Z_i is constrained to be 1. If a resource i is not used during any time period, then Z_i can be either 1 or 0. However, the presence of Z_i in the objective function ensures that if Z_i is not constrained to be 1, then it will take on a value of 0 to minimize costs. In addition, inequality (3) allows $D_{i,j}$ to be selected only once for each resource.

Inequality (4) establishes a similar if/then constraint for Y_j which takes on a value of 1 for time periods during which the fire is uncontained, and 0 otherwise.

Constraint (5) defines L_j as total line construction up to and including time period j. Constraint (6) defines N_j as a one time period lagged variable on Y_j. The lagged variable N_j is used in the place of Y_j in (1), (2), and (4) to ensure that increments of fire perimeter growth and damage are included for the time period during which fire containment is achieved, and not just time periods during which the fire is uncontained.

Example Applications

The results in this section were generated using the LINGO optimization language to encode lines (1) through (6)[3]. For illustration, six time periods and seven fire-fighting resources are used. The necessary fire behavior inputs in Table 1 (fire perimeter and area growth per hour) were generated using the FARSITE (Finney 1998) fire simulation program.

The damage caused by the fire is assumed to be $100/ha. Fire-fighting resource production rates in Table 2 are within ranges given in the National Wildfire Coordinating Group fireline handbook (1998). Values of: arrival time (ARR), suppression (COST), and fixed rental cost (PRE) vary between fires, and so were chosen to provide illustrative examples.

Given these data, the objective function (1), and constraints (2)–(6), the optimal solution dictates that the fire is contained in time period three, when the fire has a total perimeter of 1.3 km. This is accomplished by dispatching resources one, three, and four. Applying these resources incurs total fixed rental charges of $1,400 and total suppres-

Table 2. Fire-fighting resource characteristics.

Resource	Description	Arr (hr)	Cost($/hr)	Pre ($)	Prod (km/hr)
1	Dozer	2	175	300	0.36
2	Tractor plow	2.5	150	500	0.45
3	Type I crew	0.5	125	500	0.20
4	Type II crew	1	175	600	0.25
5	Engine #1	1.5	75	400	0.09
6	Engine #2	1.5	100	900	0.10
7	Engine #3	1	125	600	0.15

Table 3. Model run comparisons.

Model run	Cost	Pre	*NVC*	*C+NVC*	Fire-fighting resources used	Hour in which containment is achieved
	($)					
1. Original model inputs (damage/ha, $100)	1,425	1,400	960	3,785	1,3,4	3
2. Fire related damages $20/ha	1,375	1,000	406	2,781	2,3	5
3. Arrival times doubled (damages/ha, $100)	1,500	1,100	3,355	5,855	2,3	5

sion costs of $1,425. The optimal resource damage (NVC) incurred is $960, resulting in a total *C+NVC* of $3,785.

An important advantage of using integer programming to solve the optimal fire organization problem is that sensitivity analysis can be readily performed on model parameters to isolate those parameters that may have a significant effect on the optimal solution. Table 3 contains the results from two such sensitivity analyses on model parameters. First, we simulate the effect of a less damaging wildfire by reducing per acre fire damages to $20 from $100 (all other model inputs are left unchanged). Second, we simulate a more remote fire by doubling the arrival time of all fire-fighting resources (all other model inputs are left unchanged).

In addition to being well suited to conducting sensitivity analysis, integer programs can readily accommodate additional constraints. This is a particularly useful characteristic when modeling fire containment, as fire managers often face such constraints. For example, a fire manager may have a finite fire management budget. We simulate such a budget cap by including a constraint of $2,500 on total costs (presuppression plus suppression)(unconstrained optimal costs for this run are $2,825:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} C_i H_j D_{i,j} + \sum_{i=1}^{n} PRE_i Z_i \leq 2{,}500 \qquad (7)$$

The model can also accommodate constraints on just presuppression expenditure, a type of constraint fire managers may also face. This type of management scenario may be modeled by including the following constraint on presuppression expenditure. Presuppression expenditure is constrained not to exceed $900, while suppression expenditure is unconstrained (unconstrained presuppression costs are $1,400):

$$\sum_{i=1}^{n} PRE_i Z_i \leq 900 \qquad (8)$$

The results in Table 4 show that both constraints have only a small impact on *C+NVC*, but that the reduction in costs is achieved at the expense of a large increase in contained fire size and associated damage.

Discussion

The *C+NVC* model provides the theoretical foundation for wildfire economics. The integer program presented successfully applies this theoretical framework to a single fire event, by identifying the specific fire-fighting resources that must be deployed to attain the minimum value of *C+NVC* for the given set of model parameters.

Given this optimal solution, the model architecture is well suited to conducting partial sensitivity analysis on model parameters. This allows the user to identify those parameters that may have a significant impact on the optimal solution. For example, Table 3 compares two runs (one and three), whose model parameters are identical except that the arrival times for run three are double those of run one. This difference in arrival times has a significant impact on optimal *C+NVC*, but no impact on the optimal mix of fire-fighting resources employed. This result would indicate to the user that a more remote fire (equivalent to a doubling of arrival times) would not alter the optimal mix of fire-fighting resources employed.

The model architecture is also well suited to modeling additional constraints on the fire containment process. Table 4 illustrates the results of runs made with a total budget constraint and a constraint on just presuppression expenditure.

The example applications illustrate the general applicability and versatility of the model architecture and its capacity to model realistic fire management constraints. The model's scope could be expanded to address a broader range of wildfire management scenarios. For example, multiple fire events are often expensive and difficult to manage, and the model could be expanded to address the problem of spatially and temporally determining the optimal mix of fire-fighting resources for such situations. This could be accomplished by adding a third dimension to the decision variable $D_{i,j}$, such that $D_{i,j,k}$ would be 1 if the *i*th resource was in use during the *j*th time period at the *k*th fire, and 0 otherwise. Similarly another subscript would be added to model parameters. For example, $A_{i,k}$ would be the arrival time of the *i*th resource to the *k*th fire. The expanded model would also need an expanded series of constraints.

Table 4. Comparison of constrained and unconstrained costs.

Model run	Cost	Pre	*NVC*	*C+NVC*	Fire-fighting resources used (see Table 2)	Hour in which containment is achieved
	($)					
Costs unconstrained	1,425	1,400	960	3,785	1,3,4	3
Costs constrained not to exceed $2,500	1,375	1,000	2,030	4,405	2,3	5
Presuppression constrained not to exceed $900	1,625	800	2,030	4,455	1,2	5

This model, like other wildfire planning models, requires the use of historic fire data. These data exhibit great variability. Within the scope of the model, this variability may be addressed using sensitivity analysis, allowing the user to identify those parameters most likely to affect the optimal solution. Another approach would be to model wildfire containment stochastically.

Extending the model to include multiple fires over an extended period of time would raise issues concerning knowledge of future wildfire events. The formulation suggested would require the user to input data on a series of historic fires. These data would allow the model to optimize resource use across time. For example, resources could be held in reserve for an upcoming large wildfire. In practice, fire managers have imperfect information about future fire events, so they would not be able to plan for future fires in the way that the model could. One possible solution to this problem would be to divide a given set of wildfires into shorter time periods. The model could then be run on each of these restricted sets of fires. This approach would more closely approximate the type of decisions a fire manger must make.

Endnotes

[1] The fire intensities at which fire-fighting resources are dispatched in different geographical areas.
[2] LINGO Systems.
[3] Runs were made using a Pentium IV processor, and took between 3 sec and 1 min. to find an optimal solution.

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